1 Capítulo 55 Ejercicio

$$g^{\alpha\beta}\delta\Gamma^{\mu}_{\alpha\beta} = g^{\alpha\beta}\frac{1}{2}g^{\mu\nu}(\partial_\beta\delta g_{\nu\alpha} + \partial_\alpha\delta g_{\nu\beta} - \partial_\nu\delta g_{\alpha\beta}) \tag{1}$$

y

$$g^{\beta\mu}\delta\Gamma^{\lambda}_{\lambda\beta} = g^{\beta\mu}\frac{1}{2}g^{\lambda\nu}(\partial_\beta\delta g_{\nu\lambda} + \partial_\lambda\delta g_{\nu\beta} - \partial_\nu\delta g_{\lambda\beta}) \tag{2}$$

en la ecuación (02) haremos la siguiente operación en los índices mudos: $\beta \to \nu$, $\lambda \to \alpha$ y $\nu \to \beta$.

$$g^{\nu\mu}\frac{1}{2}g^{\alpha\beta}(\partial_\nu\delta g_{\beta\alpha} + \partial_\alpha\delta g_{\nu\beta} - \partial_\beta\delta g_{\alpha\nu}) \tag{3}$$

ahora restando (03) de (01)

$$g^{\alpha\beta}\frac{1}{2}g^{\mu\nu}(\partial_\beta\delta g_{\nu\alpha} + \partial_\alpha\delta g_{\nu\beta} - \partial_\nu\delta g_{\alpha\beta}) - g^{\nu\mu}\frac{1}{2}g^{\alpha\beta}(\partial_\nu\delta g_{\beta\alpha} + \partial_\alpha\delta g_{\nu\beta} - \partial_\beta\delta g_{\alpha\nu}) \tag{4}$$

aplicando la propiedad distributiva de la multiplicación y cancelando factores similares con un signo cambiado encontramos

$$2(g^{\alpha\beta}\frac{1}{2}g^{\mu\nu}\partial_\beta\delta g_{\mu\alpha} - g^{\alpha\beta}\frac{1}{2}g^{\mu\nu}\partial_\nu g_{\alpha\beta}) \tag{5}$$

simplificando

$$g^{\alpha\beta}g^{\mu\nu}(\partial_\beta\delta g_{\mu\nu} - \partial_\nu\delta g_{\alpha\beta}) \tag{6}$$

como α y β son índices mudos podemos escribir

$$g^{\alpha\beta}g^{\mu\nu}(\partial_\alpha\delta g_{\nu\beta} - \partial_\nu\delta g_{\alpha\beta}) \tag{7}$$

como queríamos demostrar.